Secondment Report Form

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Host Organization	Id:	
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Research Topic(s)		
	Numerical integration of Sommerfeld integrals	

ACTIVITIES DURING THE SECONDMENT

<Brief description of the main activities developed during the stay, and how they contributed to achieve your work plan goals (max. 3 pages)>

One of the most proven mathematical models to analyze antennas and scatterers embedded in planar multilayered media is based on the integral equation formulation, combined with a Galerkin method of moments (MoM). Its numerical solution calls for the fast and accurate computation of the associated Green's functions in spatial domain, commonly known as Sommerfeld integrals (SIs):

$$S_n\{G(k_\rho; z|z')\} = \int_0^\infty G(k_\rho; z|z') J_n(k_\rho \rho)\, k_\rho \, dk_\rho$$

These semi-infinite range integrals with Bessel function kernels are very difficult to calculate because of the highly oscillating and slowly decaying nature of its integrands and the singularities they possess on and/or near the real axis. Classical numerical methods for evaluating the SIs suggest splitting the integral into two parts, as shown in Fig. 1.

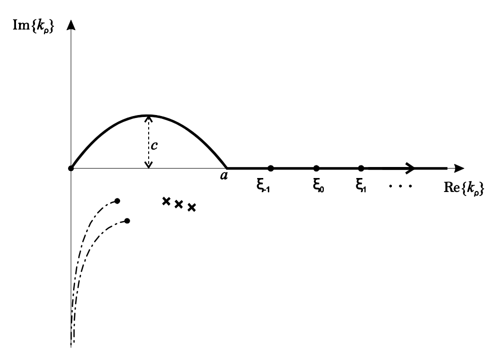

Fig. 1. Deformed integration path for the computation of Sommerfeld integrals.

The first part is joining the origin to the point a > 0, that is appropriately selected in order to ensure that there are no singularities for $Re\{k_\rho\} > a$, and is deformed into the first quadrant of the complex spectral plane. The exact shape of the initial path is not critical, and the half-sine shape with $a = k_0\left(\sqrt{n_{max} + 1}\right)$, where $n_{max} = max\left\{Re\{\varepsilon_i \mu_i\}\right\}$, $i = 1, 2, \dots, N-1$ can be used (N being the number of layers of stratified media). The maximal sine height *c* is limited by the exponential growth of the Bessel function when k_ρ is becoming complex, and is set to:

$$\frac{c}{k_0} = \begin{cases} min\left(1, \frac{1}{k_0\rho}\right), & \text{if } \rho > Re\{|z-z'|\} \\ 1, & \text{otherwise.} \end{cases}$$

But, since the path passes very near to the real-axis singularities of the spectra when $k_0\rho$ becomes large in magnitude, the proposed approach can be applied only in the case of small to moderate values of $k_0\rho$, up to about 10^2.

The remaining second part of the integral is the semi-infinite SI tail. Recently, the visitor's institution has proposed a very powerful technique for the direct integration of SI tail, based on the double-exponential (DE) quadrature formulas. Compared to the integration-then-summation approach combined with the WA method, that is proven to be one of the most versatile and efficient methods for the evaluation of SI tails, the proposed method maintains the high accuracy and error controllable nature, while reducing the overall computational cost. The method cannot be directly applied to the SI because of the aforementioned singularities. If the surface wave poles and branch point singularities are extracted from the spectral domain Green's functions, the DE quadrature formula can be directly applied to on the integration interval $(0,\infty)$, and the complete integration could be performed on the real-axis, overcoming the limitations of the deformed-path procedure.

The behavior of the spectral domain Green's function in the proximity of a surface wave pole P can be annihilated by subtracting the following spectral function:

$$c^{P,-1}(k_\rho) = 2P\left(\frac{1}{k_\rho^2 - P^2} - \frac{1}{k_\rho^2 + P^2}\right) = \frac{4P}{k_\rho^4 - P^4}$$

The corresponding spatial function can be calculated analytically:

$$C^{P,-1} = -\frac{jP}{2}\left[H_0^{(2)}(\mathrm{Pr}) - K_0(\mathrm{Pr})\right]$$

For non-homogeneous planar structures, problematic behavior around branch-point K can be annihilated by subtracting the spectral function:

$$c^K(k_\rho) = \frac{1}{K^2}\left(\sqrt{k_\rho^2 - K^2} - k_\rho + \frac{K^2}{2\sqrt{k_\rho^2 + K^2}}\right)$$

Its corresponding contribution in space domain is given by:

$$C^K(r) = \frac{K}{2\pi}\left(-\left(\frac{1}{(Kr)^3} + j\frac{1}{(Kr)^2}\right)e^{jKr} + \frac{1}{(Kr)^3} + \frac{1}{2Kr}e^{Kr}\right)$$

In this way, the problematic behaviors due to the poles and branch-point singularities is extracted in spectral-domain, leading to a very smooth spectral-domain Green's function, which can be efficiently integrated over the interval $(0,\infty)$ by directly applying DE quadrature rule. The contributions of those singularities in the space domain are analytically evaluated, as explained above.

MAIN RESULTS OF THE STAY

< List of the publications co-written (or in progress)>

The aim of this scientific mission was exchanging the experiences of the host institution technique of extracting the surface-wave poles and branch point singularities and the visitor institution approach of direct integration of SI tails using DE quadrature rule. The main goal is combining these two techniques, leading to a very robust and efficient method for the evaluation of SIs. A very detailed discussion about possible difficulties is performed, and the implementation of this hybrid method for evaluation of the Sommerfeld integrals is in progress. The joint publication is expected.

	Other(s):
Number of Publications: _____	(1)________________________________
	________________________________
Number of Documents/ Reports: _____	(2)________________________________
	________________________________
Number of Case Studies & Demonstrators: _____	(3)________________________________
	________________________________

* Attach all relevant documentation that specifies your results

FORECAST ACTIVITIES

<Are there any envisaged activities following this secondment project, new collaborations, co directed PhD, etc>

The further collaboration is expected in order to finish combining the two techniques. The expected outcome of this collaboration is future joint publication.

In order to improve CARE's secondment program, please fill this short questionnaire. Use the space at the end to expand your answers, if needed. Our aim is to improve the general experience for secondees in future.

Disagree ≺ ≻ Agree

GENERAL				
My objectives were achieved.	**1** □	**2** □	**3** □	**4** √
The research topics were relevant to my work.	**1** □	**2** □	**3** □	**4** √
I benefited from being part of a wider research culture.	**1** □	**2** □	**3** □	**4** √

HOST ORGANIZATION				
I am satisfied with the quality and quantity of supervision I received.	**1** □	**2** □	**3** □	**4** √
I had access to adequate resources to support my research..	**1** □	**2** □	**3** □	**4** √

SECONDMENT PROGRAM				
I would recommend this secondment programme to others.	**1** □	**2** □	**3** □	**4** √
I believe the skills I have learned will help me to improve my job/research.	**1** □	**2** □	**3** □	**4** √
I would apply to another programme similar to CARE.	**1** □	**2** □	**3** □	**4** √
In general, how would you classify the CARE Secondment Programme?	**1** □	**2** □	**3** □	**4** √

Other questions/comments to be potentially considered: __________________________________

SIGNATURES

Candidate Ruzica Golubovic Niciforovic

Date: 2011/01/18
(year/month/day)

Signature ____