

Numerical MoM treatment of cloak with cyclic symmetry

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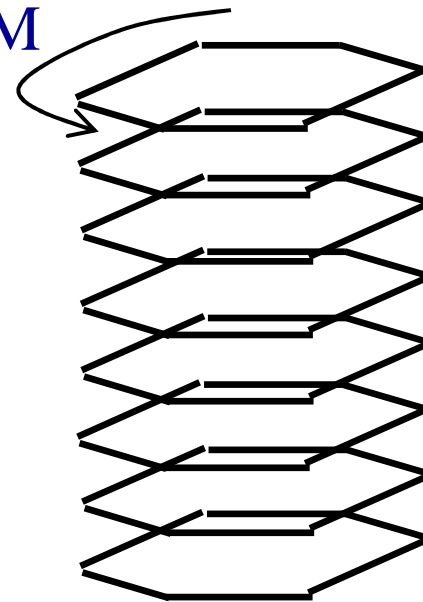
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Objective: A fast method to analyze the scattering characteristics from a cloak exploiting its azimuthal symmetry and periodicity along the axis of the cloak.

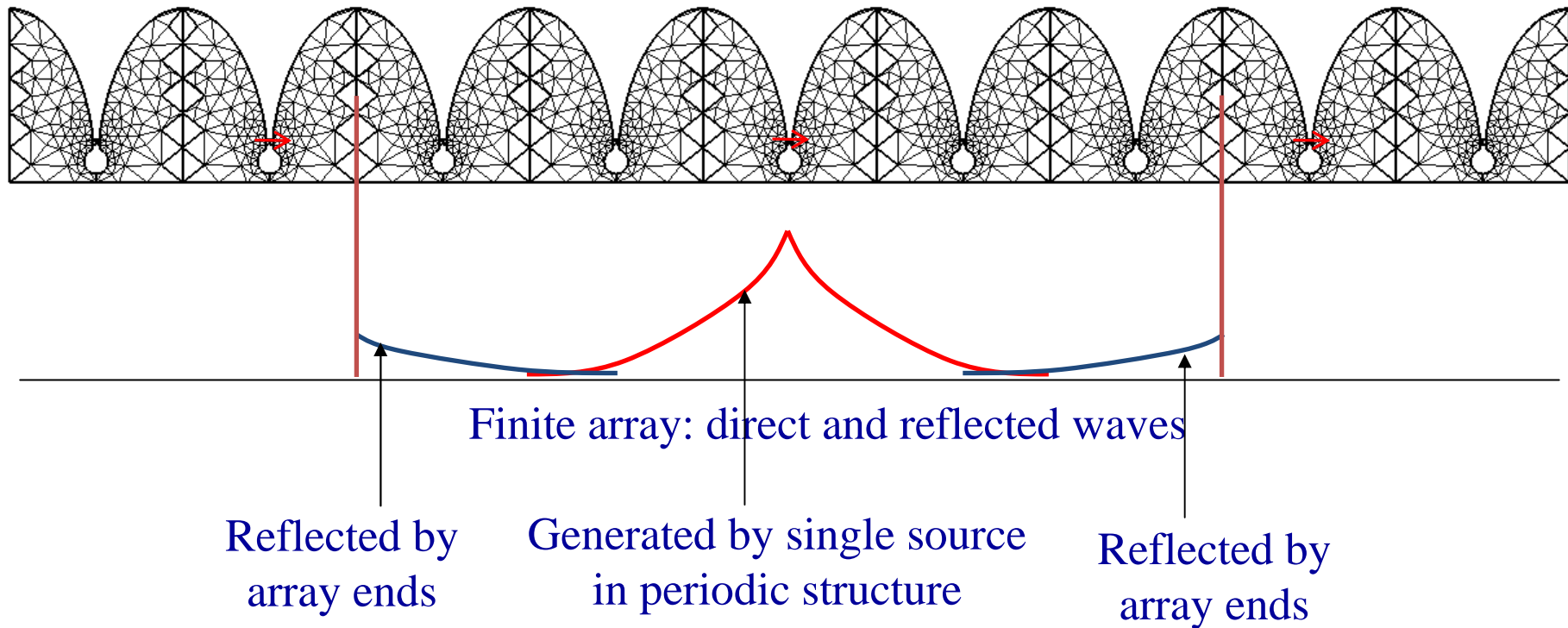
- To exploit the cyclic symmetry the **Array Scanning Method** is used.
- Along the axis of the cylinder the **Method-of-Moments** while exploiting the **1D periodic scalar Green's functions** and the mixed-potential integral equation is used.

ASM



Infinite with
 $\psi_z = 0$

Wave phenomenology in finite arrays



- The current on a given point can be regarded as progressive waves launched by the excited element and reflected by the ends of the array.

Array Scanning Method (1/2)

$$\underbrace{I(m)}_{\text{Current at ant. } m \text{ for ant. } 0 \text{ excited}} = \frac{1}{2\pi} \int_0^{2\pi} \underbrace{I^\infty(\psi)}_{\text{Infinite-array solution for phase shift } \psi \text{ between elements}} e^{-j m \psi} d\psi \quad (\text{B. Munk et al., 1979})$$

Current at ant. m
for ant. 0 excited



Infinite-array solution for phase
shift ψ between elements

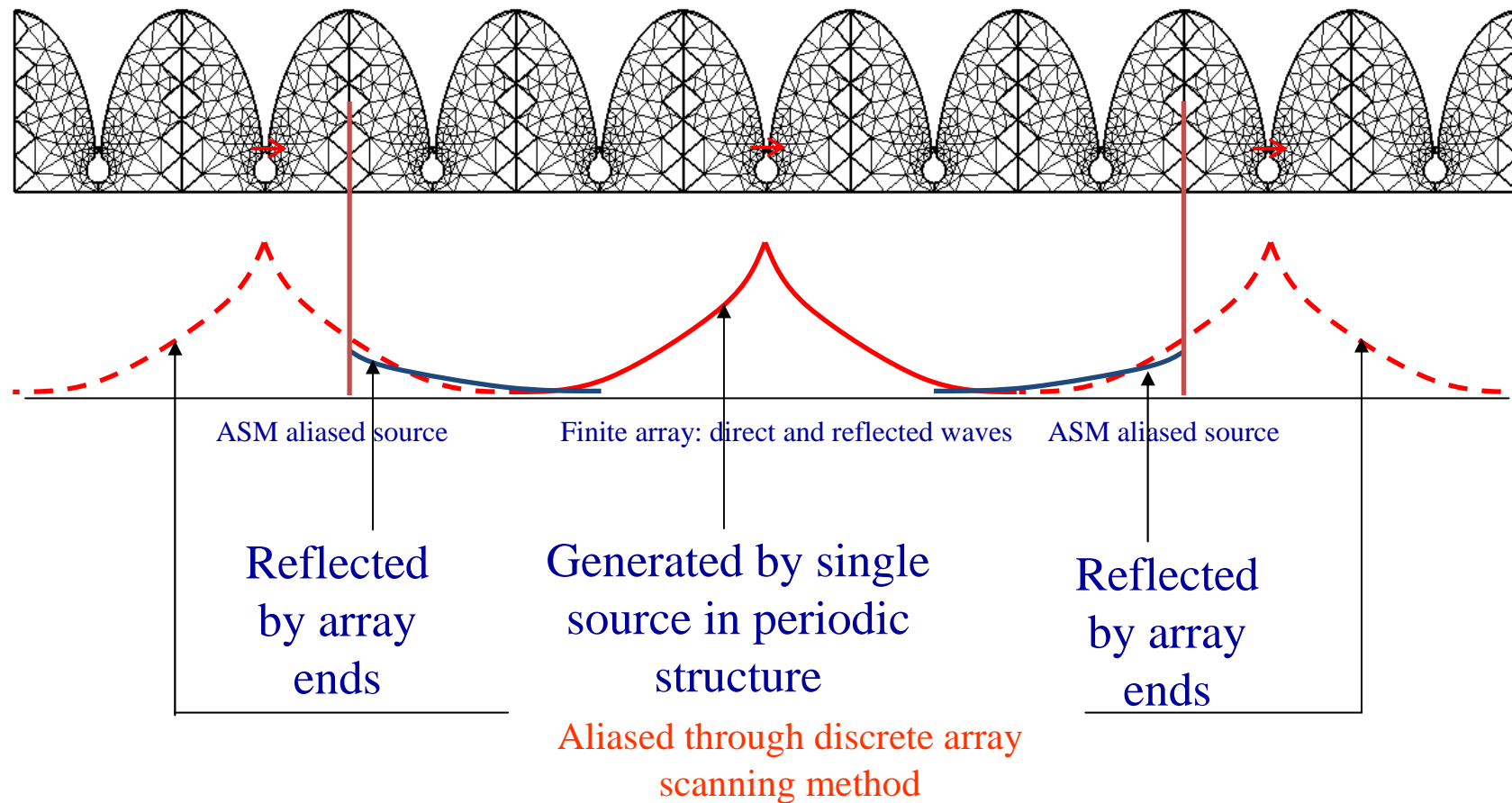
$$I(m) \simeq \frac{1}{N} \sum_{p=0}^{N-1} I^\infty(\psi_p) e^{-j m \psi_p}$$

Aliasing:

Repetition of the source
every N elements

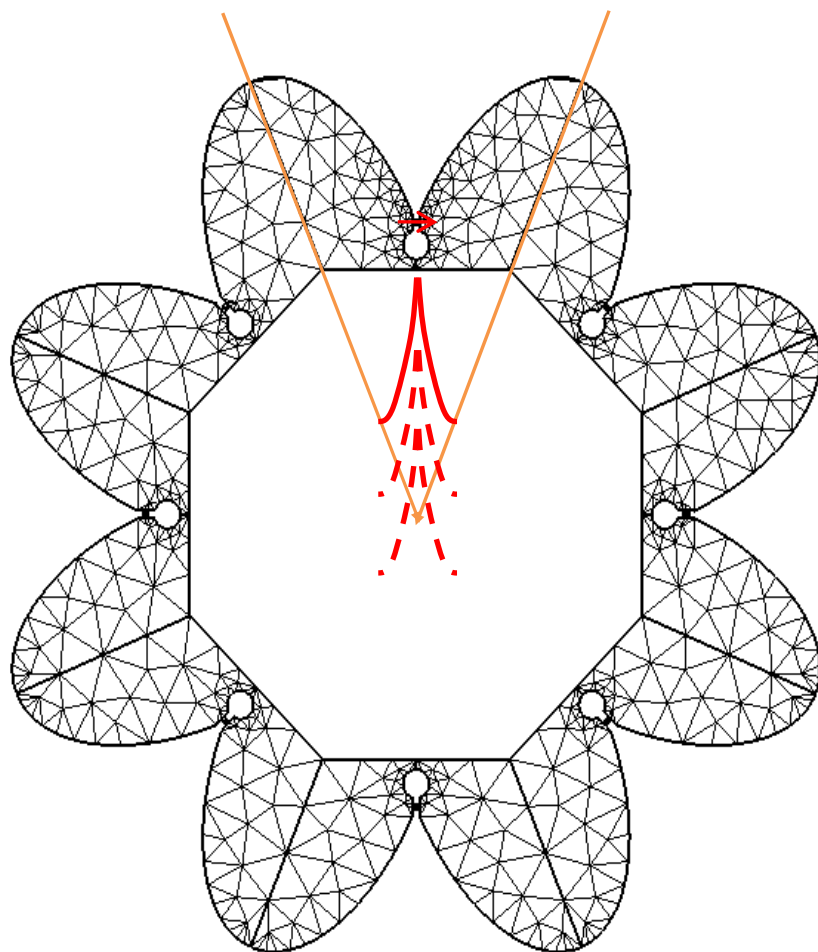
$$\psi_p = 2\pi p/N$$

Array Scanning Method (2/2)



- If Array Scanning Method is implemented with the help of finite summation, the source is repeated. (see figure auxiliary peaks)

ASM applied to Circular Arrays (1/2)



- Repeated source every N elements, i.e. always on the same element in N -element circular array : with the aliased source, the exact solution is obtained !

ASM applied to Circular Arrays (2/2)

Method of Moments
(N*M)x(N*M)
system of equations

$$I(m) = \frac{1}{N} \sum_{p=0}^{N-1} I^{\infty}(\psi_p) e^{-jm\psi_p}$$

$$\psi_p = p \frac{2\pi}{N} \text{ with } (0 < p < N-1)$$

N Reduced (MxM) systems.

$$\begin{bmatrix} Z_{11} & Z_{12} & \dots & \dots & Z_{1N} \\ Z_{1N} & Z_{11} & Z_{12} & \dots & Z_{1N-1} \\ Z_{1N-1} & Z_{1N} & Z_{11} & \dots & Z_{1N-2} \\ \dots & \dots & \dots & \dots & \dots \\ Z_{12} & Z_{13} & \dots & Z_{1N} & Z_{11} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix}$$

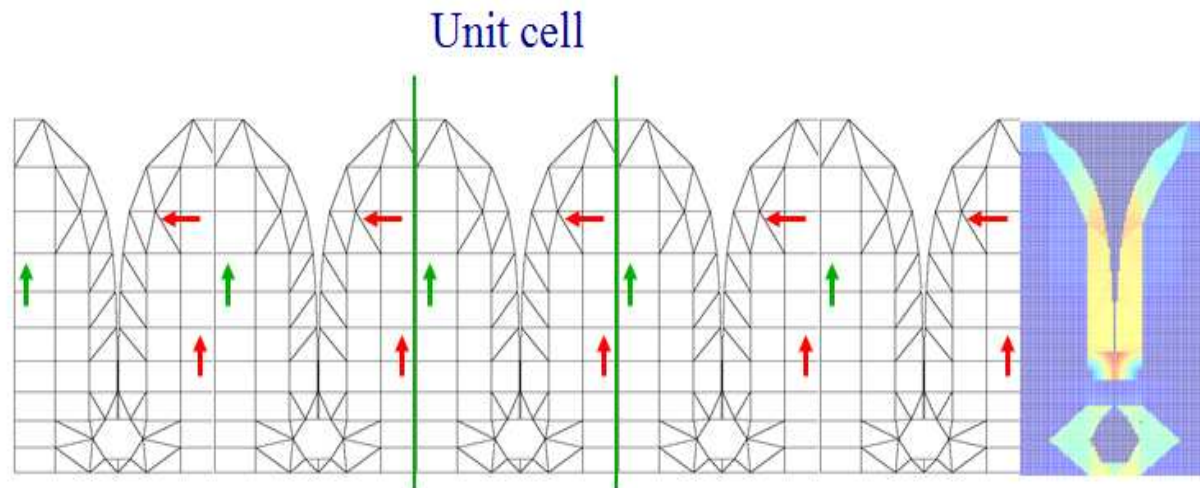
ASM solution:

$$[Z_c][I^{\infty}(\psi_p)] = [V(\psi_p)]$$

$$Z_c = Z[C_1 C_2 \dots C_{N-1} C_N]^T$$

$$C_p = U(N) * e^{jm\psi_p}$$

Infinite Linear Array approach

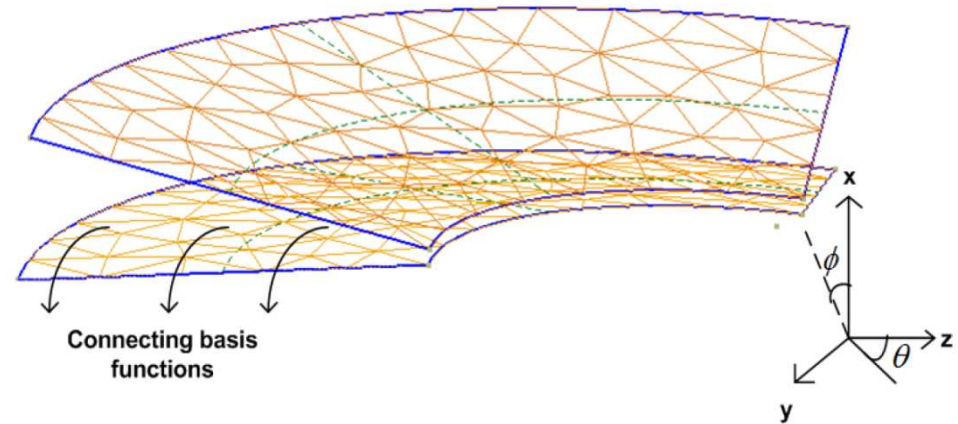
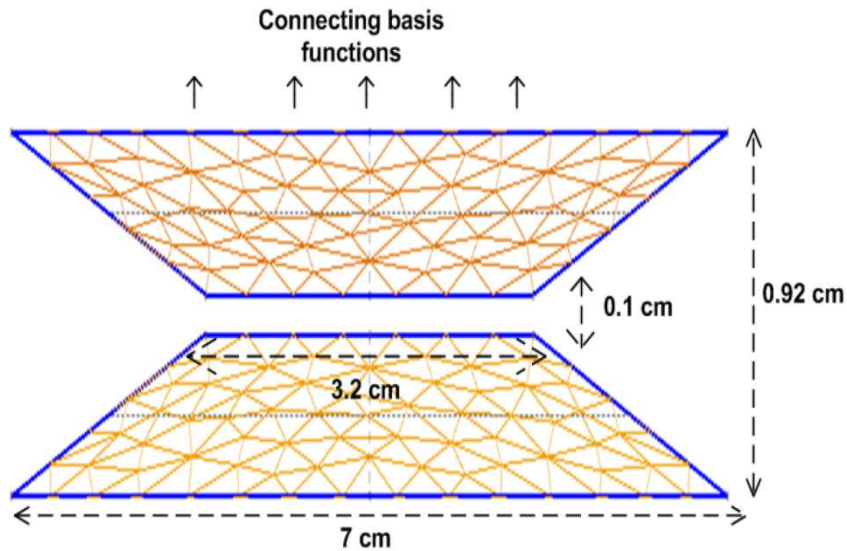


Scalar
Green's
function G

$$\sum_{q=-\infty}^{\infty} \frac{e^{-j k_{yq} y}}{4 j d_y} H_0^{(2)}(k_{\rho q} R_m) e^{-j m k_{x0} d_x}$$

Cylindrical waves

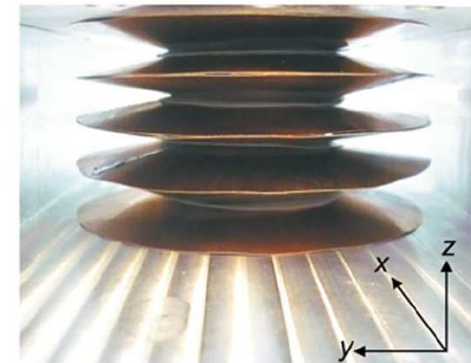
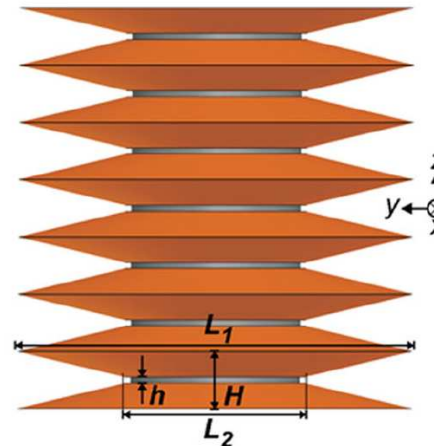
Numerical Simulation



- Order **N** saving obtained in terms of **memory**.

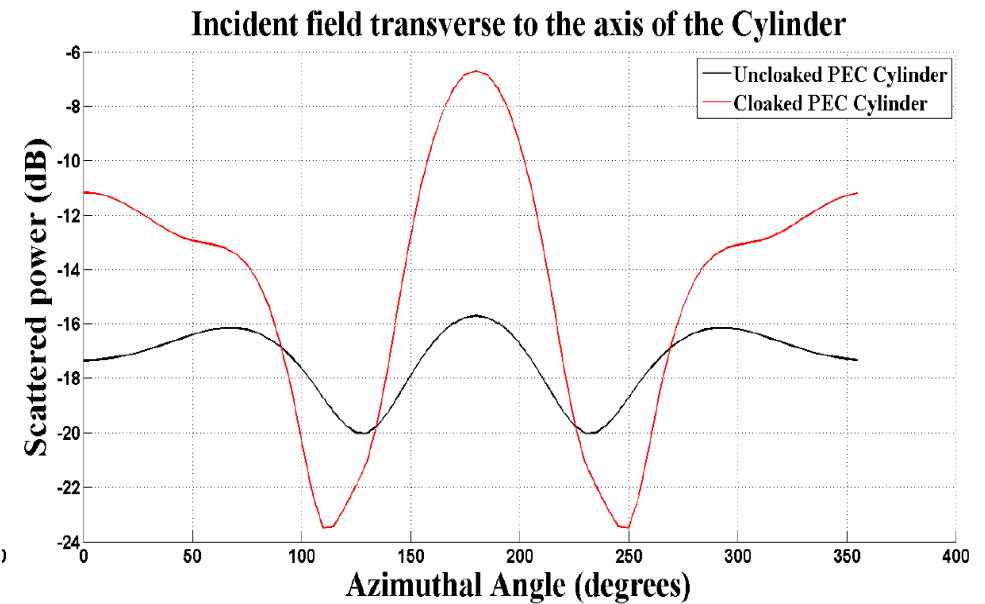
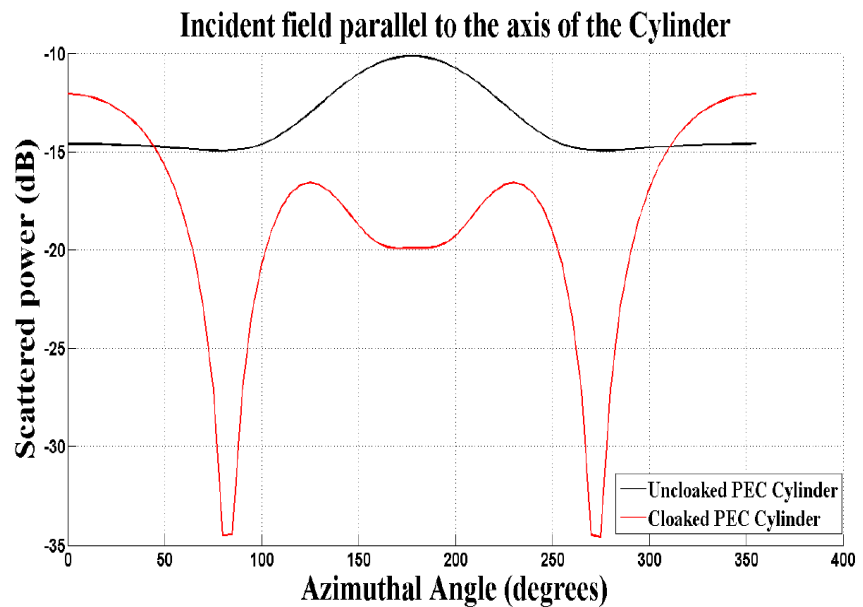
- Calculation **time** reduced by **N²**

N is the number of cyclic sectors.



"Broadband Electromagnetic Cloaking of Long Cylindrical Objects", S.Tretyakov, P.Alitalo, O.Luukkonen, C.Simovski, PRL 103, 103905, September, 2009

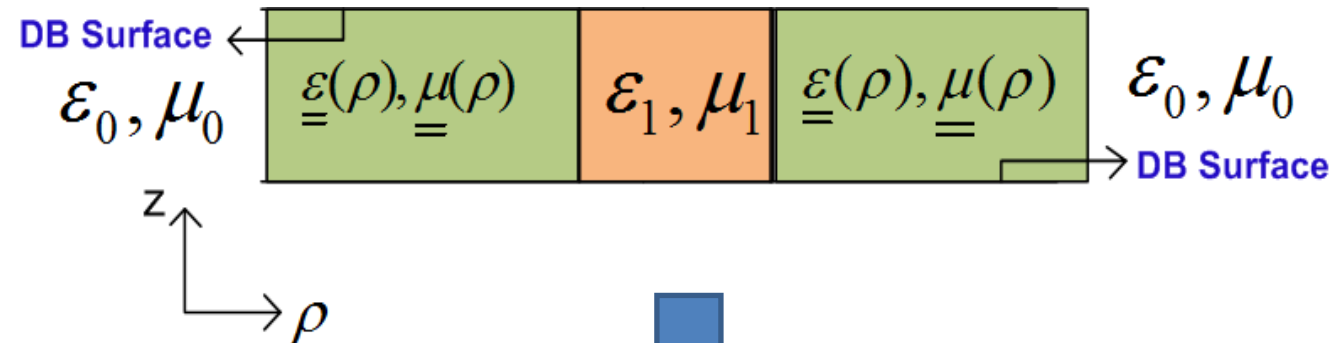
Scattered power



— Uncloaked PEC Cylinder
— Cloaked PEC Cylinder

Using coordinate transformation to obtain cloak in free space

**Original
space**

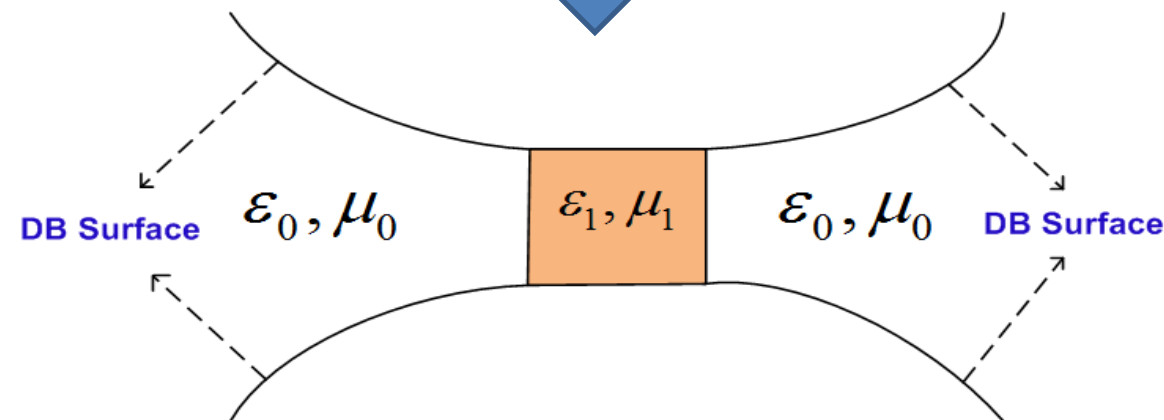


DB Surfaces:

$$\hat{n} \cdot \vec{D} = 0 \quad \hat{n} \cdot \vec{B} = 0$$

**Coordinate
Transformation**

**Real
space**



Attempt to arrive at cloak in free space from volumetric anisotropic medium

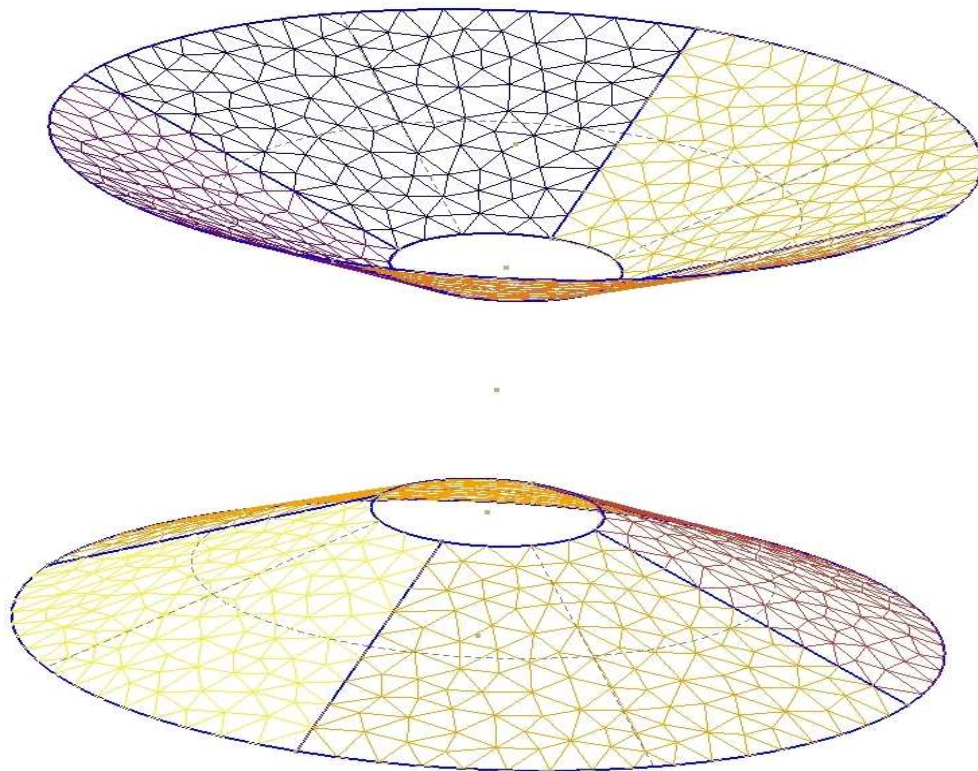
$$\varepsilon_0 \underline{\underline{I}} = \det(\underline{\underline{M}}) \underline{\underline{M}}^{-1} \underline{\underline{\varepsilon}}(r) (\underline{\underline{M}}^T)^{-1}$$

$$\underline{\underline{M}} = \left\{ \frac{\partial x'_i}{\partial x_j} \right\}_{i,j=1,3} ; \underline{\underline{M}}^{-1} = \left\{ \frac{\partial x_i}{\partial x'_j} \right\}_{i,j=1,3}$$

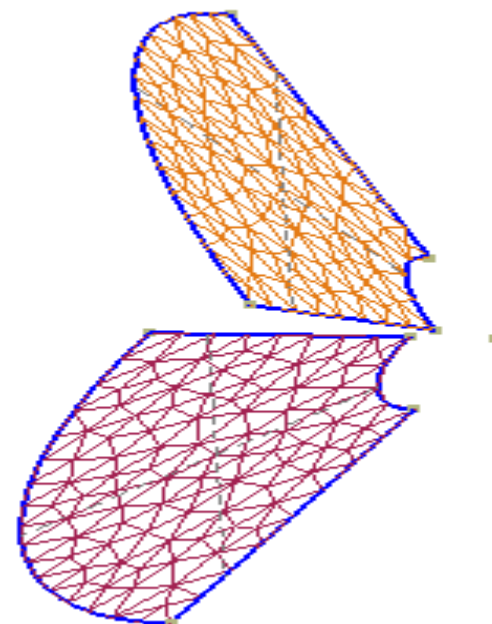
- Using the property of the *covariant* and the *contravariant* vectors from the coordinate transformation it is confirmed that the DB boundary conditions are fulfilled in the arrival space.
- The distance between the plates at the inner boundary of the cloak will be at cutoff and not allow any propagation.
- The waves will then propagate around the cloaked object.

Modeling of the free space cloak

(Work in progress)



A unit cell along the
cylinder axis



A sector along the
azimuthal direction

Thank you!

